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Comment on the Alday-Maldacena solution in calculating scattering amplitude via AdS/CFT

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Abstract: Following the recent proposal of Alday and Maldacena to obtain the strong coupling scattering amplitude in $\mathcal{N} = 4$ SYM via AdS/CFT, we point out that a unique solution can be obtained by imposing all the Virasoro constraints. In the case of four-gluon scattering, this solution is identical to the Alday-Maldacena solution, which is in accordance with the ansatz of Bern, Dixon and Smirnov. This also solves the moduli space problem of the four-point solution in a recent paper of Mironov, Morozov and Tomaras.

Keywords: Strong Coupling Expansion, AdS-CFT Correspondence, Supersymmetric gauge theory.

Recently Alday and Maldacena proposed a novel method to calculate planar gluon scattering amplitudes at strong coupling in $\mathcal{N} = 4$ SYM by using AdS/CFT duality [1]. At leading order the calculation is reduced to finding the minimal area of a string with a light-like boundary.

On the other hand, in $\mathcal{N} = 4$ SYM, an ansatz for the all order form of the n gluon MHV scattering amplitudes has already been given by Bern, Dixon and Smirnov [2]. This ansatz (the BDS ansatz) was supposed to be valid at both weak and strong coupling. In the weak coupling regime, it has been verified for the four-point amplitude up to five-loop and five-point amplitude up to two-loop [3]–[7].

By using their proposal [1], Alday and Maldacena computed the explicit form of the amplitude for the scattering of four gluons and found precise agreement with the BDS ansatz to the leading order of strong coupling. Inspired by this new correspondence, there have appeared a number of closely related works and generalizations in [8]–[30]. In particular, an important quantity in the BDS ansatz, the one-loop MHV n-gluon amplitude, can be written as a double contour integral along a polygonal Wilson loop Π, which is defined by the external gluons momenta [11](see also [10]):

$$M_n^{(1)} = \oint_\Pi \oint_\Pi \frac{dy^\mu dy'_\mu}{[(y-y')^2]^{1+\epsilon}} \tag{1}$$

By the proposal of Alday and Maldacena [1], this geometrical integral should be identified with another geometric quantity: the minimal area of a string in AdS_5 which is bounded by the same polygon (see [16] for more details).

To realize the proposal of Alday and Maldacena, it is essential to find the classical string solution with given boundary conditions. In [1], the solution of four-gluon scattering was found by doing conformal transformations to a cusp solution, or by trial and error. For the general multi-gluon scattering, it's more difficult to find solutions.[1] Due to the lack of a general method to solve the complicated equations, it is also not clear whether the solution is unique or not.

In a recent paper [16], Mironov, Morozov and Tomaras solved the sigma-model equations of motion in the case of four-gluon scattering by using a special ansatz. Surprisingly, the solution was found to have a moduli space $\{z_a, \phi\}$ [16], and moreover, the regularized minimal area is also moduli dependent. This raises a problem: which solution in the moduli space is the 'right' solution that corresponds to the unique scattering amplitude? In [16], the authors suggested that the Alday-Maldacena solution could be considered as a minimum of the regularized action in the moduli space.

In this paper, we point out that a unique solution can be obtained by imposing all the Virasoro constraints.[2] We will show this explicitly in the case of four-gluon scattering,

[1]There have been discussions on the solutions of the large n-point case in [22], and 6-point and 8-point case in [24]. The interesting dressing method for finding new possible solutions was discussed in [15].

[2]The Virasoro constraints were also considered in [26] when the relation between the σ-model action and Nambu-Goto action was discussed. For the Nambu-Goto action, due to the reparametrization invariance, we can always choose a parametrization to give the same equations of motion plus the Virasoro constraints as that from the string σ-model action.

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where the moduli space variables $\{z_a, \phi\}$ in [16] can be fixed uniquely by the Virasoro constraints. This is supposed to be true in the cases of general multi-gluon scattering.

We first give a short review of the solution in [16]. We will follow closely the notations used in [16].

The string σ-model action is

$$S[X,g] = \int d^2u \ \sqrt{g} \ g^{ij} \ G_{MN}\partial_i X^M \partial_j X^N, \tag{2}$$

where the world-sheet metric $g_{ij}(u_1, u_2)$ is taken to be Euclidean.

In conformal gauge and for the AdS_5 target space, the string σ-model action takes the following form

$$S = \int d^2u \ \frac{(\vec{\partial} r)^2 + (\vec{\partial}\mathbf{y})^2}{r^2}. \tag{3}$$

The bold font is for 4d vectors in the target space, while arrow is used for 2d vectors on the world-sheet.

The equations of motion are

$$\vec{\partial}\left(\frac{\vec{\partial} r}{r^2}\right) = -\frac{L}{r}, \quad \vec{\partial}\left(\frac{\vec{\partial}\mathbf{y}}{r^2}\right) = 0, \tag{4}$$

$$L = \frac{(\vec{\partial} r)^2 + (\vec{\partial}\mathbf{y})^2}{r^2}. \tag{5}$$

The solution should also satisfy the Virasoro constraints, i.e. $\delta_g S[X,g] = 0$, which in conformal gauge read

$$(\partial_1 r)^2 - (\partial_2 r)^2 + (\partial_1 \mathbf{y})^2 - (\partial_2 \mathbf{y})^2 = 0, \tag{6}$$

$$\partial_1 r \partial_2 r + \partial_1 \mathbf{y} \partial_2 \mathbf{y} = 0. \tag{7}$$

In coordinate $z = 1/r, \mathbf{v} = \mathbf{y}/r$, the equations of motion take the following form

$$\Delta z = zL, \quad \Delta \mathbf{v} = \mathbf{v}L, \tag{8}$$

$$z^2 L - (\vec{\partial} z)^2 = (z\vec{\partial}\mathbf{v} - \mathbf{v}\vec{\partial} z)^2, \tag{9}$$

where $\Delta \equiv \partial^2/\partial u_1^2 + \partial^2/\partial u_2^2$ is the world-sheet Laplacian. And the Virasoro constraints take the following form

$$(\partial_1 z)^2 - (\partial_2 z)^2 + (z\partial_1\mathbf{v} - \mathbf{v}\partial_1 z)^2 - (z\partial_2\mathbf{v} - \mathbf{v}\partial_2 z)^2 = 0, \tag{10}$$

$$(\partial_1 z)(\partial_2 z) + (z\partial_1\mathbf{v} - \mathbf{v}\partial_1 z)(z\partial_2\mathbf{v} - \mathbf{v}\partial_2 z) = 0. \tag{11}$$

For $L = const$, an ansatz of the solution is given in [16] as

$$z = \sum_{a=1}^{n} z_a e^{\vec{k}_a \cdot \vec{u}}, \quad \mathbf{v} = \sum_{a=1}^{n} \mathbf{v}_a e^{\vec{k}_a \cdot \vec{u}}, \tag{12}$$

where n is the number of external gluons. And the boundary conditions are given as

$$\Delta_a \mathbf{y} = \frac{\mathbf{v}_{a+1}}{z_{a+1}} - \frac{\mathbf{v}_a}{z_a} = \mathbf{p}_a, \tag{13}$$

where $\mathbf{p}_a$ are the n external momenta.

It is easy to see that the ansatz eq. (12) satisfies eq. (8) if $\vec{k}_a^2 = L$. Nontrivial equations are eqs. (9)–(11). By substitution of eq. (12), eq. (9) takes the following form

$$\sum_{a,b} z_a z_b \Big(L - (\vec{k}_a \cdot \vec{k}_b)\Big) E_{a+b} - \sum_{a<b,\ c<d} (\mathcal{P}_{ab}\mathcal{P}_{cd})(\vec{k}_{ab} \cdot \vec{k}_{cd}) E_{a+b+c+d} = 0, \tag{14}$$

and the Virasoro constraints eqs. (10)–(11) take the following form

$$\sum_{a,b} z_a z_b (k_a^1 k_b^1 - k_a^2 k_b^2) E_{a+b} + \sum_{a<b,\ c<d} (\mathcal{P}_{ab}\mathcal{P}_{cd})(k_{ab}^1 k_{cd}^1 - k_{ab}^2 k_{cd}^2) E_{a+b+c+d} = 0, \tag{15}$$

$$\sum_{a,b} z_a z_b (k_a^1 k_b^2 + k_a^2 k_b^1) E_{a+b} + \sum_{a<b,\ c<d} (\mathcal{P}_{ab}\mathcal{P}_{cd})(k_{ab}^1 k_{cd}^2 + k_{ab}^2 k_{cd}^1) E_{a+b+c+d} = 0, \tag{16}$$

where all the summations are from 1 to n, and

$$\begin{aligned} E_{a_1+\ldots+a_m} &= e^{(\vec{k}_{a_1}+\ldots+\vec{k}_{a_m})\cdot\vec{u}}, \quad \vec{k}_{ab} = \vec{k}_a - \vec{k}_b, \quad \vec{k} = (k^1, k^2), \\ \mathcal{P}_{ab} &= z_a \mathbf{v}_b - z_b \mathbf{v}_a = z_a z_b (\mathbf{p}_a + \mathbf{p}_{a+1} + \ldots + \mathbf{p}_{b-1}). \end{aligned} \tag{17}$$

Now we try to solve these equations. We first consider eq. (14). This equation consists of a summation of a series of independent E-functions ($E_{a+b+\ldots}$). So solving this equation is equivalent to requiring the vanishing of the coefficient of each independent E-function. Let's first study the terms of $E_{2a+(a-1)+(a+1)}$. Eq. (14) requires that

$$\begin{aligned} 0 &= \big(\mathcal{P}_{(a-1)a}\mathcal{P}_{a(a+1)}\big)\left(\vec{k}_{(a-1)a} \cdot \vec{k}_{a(a+1)}\right) E_{2a+(a-1)+(a+1)} \\ &= z_a^2 z_{a-1} z_{a+1} (2\mathbf{p}_{a-1}\mathbf{p}_a)\left(\vec{k}_{(a-1)a} \cdot \vec{k}_{a(a+1)}\right) E_{2a+(a-1)+(a+1)}, \quad \text{for all } a = 1, 2, \ldots, n \end{aligned} \tag{18}$$

where $n+1 = 1$ by cyclicity. Since $2\mathbf{p}_{a-1}\mathbf{p}_a = (\mathbf{p}_{a-1} + \mathbf{p}_a)^2 \neq 0$, the above equations are equivalent to

$$\vec{k}_{(a-1)a} \cdot \vec{k}_{a(a+1)} = (\vec{k}_{a-1} - \vec{k}_a) \cdot (\vec{k}_a - \vec{k}_{a+1}) = 0, \quad \text{for all } a = 1, 2, \ldots, n. \tag{19}$$

For $\vec{k}_a^2 = L = const$, these conditions can be all satisfied only at $n = 4$, where the four $\vec{k}$-vectors point along the diagonals of a rectangle. This indicates that the ansatz eq. (12) can *not* be applied to $n > 4$ cases directly.[3] We will only consider the 4-point case.

We can take the $\vec{k}$-vectors generally as

$$\begin{aligned} \vec{k}_1 &= \sqrt{L}(\cos\phi_1, \sin\phi_1), & \vec{k}_2 &= \sqrt{L}(\cos\phi_2, -\sin\phi_2), \\ \vec{k}_3 &= \sqrt{L}(-\cos\phi_1, -\sin\phi_1) = -\vec{k}_1, & \vec{k}_4 &= \sqrt{L}(-\cos\phi_2, \sin\phi_2) = -\vec{k}_2. \end{aligned} \tag{20}$$

[3] We can also study the interesting 3-point case. The scattering amplitude of three on-shell gluons is identically zero. In Alday and Maldacena's proposal, this can be understood by noticing that three lightlike lines can not constitute a triangle. But if one gluon is off-shell, it is possible to find a solution for eq. (14) under the ansatz eq. (12). However, this solution has a non-analytic point, and even worse, it is incompatible with one of the Virasoro constraints.

The parameter L is inessential, due to the scaling reparametrization invariance of (u_1, u_2). On the other hand, the parameters $\{\phi_1, \phi_2\}$ are important, since different values of $\{\phi_1, \phi_2\}$ can correspond to physically inequivalent solutions.

After substitution of eq. (20) for $\vec{k}$-vectors, eq. (14) can be collected as

$$0 = (1 - z_1 z_3 s - z_2 z_4 t)\left[\sin^2\left(\frac{\phi_1+\phi_2}{2}\right)(z_1 z_2 E_{1+2} + z_3 z_4 E_{3+4}) \right.$$
$$\left. + \cos^2\left(\frac{\phi_1+\phi_2}{2}\right)(z_1 z_4 E_{1+4} + z_2 z_3 E_{2+3}) + (z_1 z_3 + z_2 z_4) E_0\right], \tag{21}$$

where $s = (p_1 + p_3)^2,\ t = (p_2 + p_3)^2$ are the Mandelstam variables. The coefficients of the remaining five independent E-functions have a common factor, so the equation can be (and only be) solved by requiring this factor to vanish, i.e.

$$z_1 z_3 s + z_2 z_4 t = 1. \tag{22}$$

From this relation, we see that there is still much freedom of choosing z_a, while $\{\phi_1, \phi_2\}$ are totally unfixed.

Next we consider the Virasoro constraints eq. (15) and eq. (16). Similar to solving eq. (14), we substitute eq. (20) for $\vec{k}$-vectors, and collect the terms for independent E-functions. Then eq. (15) gives

$$\begin{aligned}
&\{\cos(2\phi_1) z_1 z_3 [1 - 2 z_1 z_3 s - (s+t) z_2 z_4] + \cos(2\phi_2) z_2 z_4 [1 - 2 z_2 z_4 t - (s+t) z_1 z_3]\} 2E_0 \\
&-[\cos(2\phi_1) z_1 z_3 s + \cos(2\phi_2) z_2 z_4 t + \cos(\phi_1 - \phi_2)(1 - z_1 z_3 s - z_2 z_4 t)] 2(z_1 z_2 E_{1+2} + z_3 z_4 E_{3+4}) \\
&-[\cos(2\phi_1) z_1 z_3 s + \cos(2\phi_2) z_2 z_4 t - \cos(\phi_1 - \phi_2)(1 - z_1 z_3 s - z_2 z_4 t)] 2(z_1 z_4 E_{1+4} + z_2 z_3 E_{2+3}) \\
&-\{\cos(2\phi_1) - [\cos(2\phi_1) - \cos(2\phi_2)] z_2 z_4 t\}(z_1^2 E_{1+1} + z_3^2 E_{3+3}) \\
&-\{\cos(2\phi_2) + [\cos(2\phi_1) - \cos(2\phi_2)] z_1 z_3 s\}(z_2^2 E_{2+2} + z_4^2 E_{4+4}) = 0,
\end{aligned} \tag{23}$$

and eq. (16) gives

$$\begin{aligned}
&\{\sin(2\phi_1) z_1 z_3 [1 - 2 z_1 z_3 s - (s+t) z_2 z_4] - \sin(2\phi_2) z_2 z_4 [1 - 2 z_2 z_4 t - (s+t) z_1 z_3]\} 2E_0 \\
&-[\sin(2\phi_1) z_1 z_3 s - \sin(2\phi_2) z_2 z_4 t + \sin(\phi_1 - \phi_2)(1 - z_1 z_3 s - z_2 z_4 t)] 2(z_1 z_2 E_{1+2} + z_3 z_4 E_{3+4}) \\
&-[\sin(2\phi_1) z_1 z_3 s - \sin(2\phi_2) z_2 z_4 t - \sin(\phi_1 - \phi_2)(1 - z_1 z_3 s - z_2 z_4 t)] 2(z_1 z_4 E_{1+4} + z_2 z_3 E_{2+3}) \\
&-\{\sin(2\phi_1) - [\sin(2\phi_1) + \sin(2\phi_2)] z_2 z_4 t\}(z_1^2 E_{1+1} + z_3^2 E_{3+3}) \\
&+\{\sin(2\phi_2) - [\sin(2\phi_1) + \sin(2\phi_2)] z_1 z_3 s\}(z_2^2 E_{2+2} + z_4^2 E_{4+4}) = 0.
\end{aligned} \tag{24}$$

We first consider the terms of E_{a+a} in the above two equations. The vanishing of their coefficients is equivalent to the following relations

$$\begin{aligned}
z_1 z_3 s &= \frac{-\cos(2\phi_2)}{\cos(2\phi_1) - \cos(2\phi_2)} = \frac{\sin(2\phi_2)}{\sin(2\phi_1) + \sin(2\phi_2)}, \\
z_2 z_4 t &= \frac{\cos(2\phi_1)}{\cos(2\phi_1) - \cos(2\phi_2)} = \frac{\sin(2\phi_1)}{\sin(2\phi_1) + \sin(2\phi_2)},
\end{aligned} \tag{25}$$

from which we get a relation for $\{\phi_1, \phi_2\}$,

$$0 = \sin(2\phi_1)\cos(2\phi_2) + \cos(2\phi_1)\sin(2\phi_2) = \sin[2(\phi_1 + \phi_2)]. \tag{26}$$

Physically we require that $\phi_1 + \phi_2 \neq 0, \pi$, so the above equation is solved by

$$\phi_1 + \phi_2 = \frac{\pi}{2} \, . \tag{27}$$

This also gives that $\sin(2\phi_1) = \sin(2\phi_2)$ and $\cos(2\phi_1) = -\cos(2\phi_2)$. Substituting this back into eq. (25), we get another relation[4]

$$z_1 z_3 s = z_2 z_4 t = \frac{1}{2} \, , \tag{28}$$

which also solves eq. (22) that we have got from solving the equations of motion.

By using eq. (27) and eq. (28), we find that all the coefficients of other E-functions in eq. (23) and eq. (24) also vanish. Therefore, eq. (27) and eq. (28) are our final constraints on the solution.

It may be a little surprising that all the three complicated equations eq. (21), eq. (23) and eq. (24) lead to only two relations. Under the special ansatz eq. (12), we actually have more equations (every independent E-function gives an equation) than the freedom of the solution, i.e. the solution is overdetermined by the equations of motion and the Virasoro constraints. Generally, there would be no solution under this ansatz, such as for $n > 4$ cases.

It seems that we still have freedom to choose the value of ϕ_1 (or ϕ_2) in eq. (27), and also have freedom to choose the value of z_1 (or z_3) and z_2 (or z_4) in eq. (28). However, all these solutions are equivalent to each other due to two kinds of reparametrization invariance of (u_1, u_2). By rotational reparametrization invariance, it's easy to see that only the sum of ϕ_1 and ϕ_2 is physically important; while the translational reparametrization invariance tells us that the freedom in z_a is trivial, which we will show explicitly below. Besides an inessential shift, the vectors $\mathbf{v}_a$ can also be fixed by the boundary conditions eq. (13) with given z_a. So the solution is actually unique.

Let's give the explicit form of the solution for r. By using eq. (27) and eq. (28), we can write z generally as

$$z = z_1 e^{\vec{k}_1 \cdot \vec{u}} + \frac{1}{2 s z_1} e^{-\vec{k}_1 \cdot \vec{u}} + z_2 e^{\vec{k}_2 \cdot \vec{u}} + \frac{1}{2 t z_2} e^{-\vec{k}_2 \cdot \vec{u}} . \tag{29}$$

By choosing a constant world-sheet vector $\vec{\delta}$ which satisfies

$$e^{\vec{k}_1 \cdot \vec{\delta}} = \sqrt{2s}\; z_1 \, , \quad e^{\vec{k}_2 \cdot \vec{\delta}} = \sqrt{2t}\; z_2 \, , \tag{30}$$

we can make a translational transformation: $\vec{u'} = \vec{u} + \vec{\delta}$. Then eq. (29) reads

$$z = \frac{1}{\sqrt{2s}} \left(e^{\vec{k}_1 \cdot \vec{u'}} + e^{-\vec{k}_1 \cdot \vec{u'}} \right) + \frac{1}{\sqrt{2t}} \left(e^{\vec{k}_2 \cdot \vec{u'}} + e^{-\vec{k}_2 \cdot \vec{u'}} \right) . \tag{31}$$

Furthermore, we can set $\phi_1 = \phi_2 = \pi/4$ by making a rotational transformation of (u_1, u_2), and also we can set $L = 2$ by making a scaling transformation of (u_1, u_2). Then we have $k_1 = (+1, +1)$ and $k_2 = (+1, -1)$. After these transformations, we can write z as

$$z = \left(\frac{1}{\sqrt{2s}} + \frac{1}{\sqrt{2t}} \right) 2 \cosh u'_1 \cosh u'_2 + \left(\frac{1}{\sqrt{2s}} - \frac{1}{\sqrt{2t}} \right) 2 \sinh u'_1 \sinh u'_2 \, . \tag{32}$$

[4] In the ansatz eq. (12) of the solution, we don't require $s, t > 0$, which corresponds to spacelike momentum transfer. However, the constraint eq. (28) infers it has to be so. Otherwise, if $z_1 z_3 < 0$ or $z_2 z_4 < 0$, there will be singular points on the boundary for the solutions, which is physically inconsistent.

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The unique solution for r is then

$$r = \frac{1}{z} = \frac{a}{\cosh u_1' \cosh u_2' + b \sinh u_1' \sinh u_2'}\,, \tag{33}$$

where

$$a = \frac{\sqrt{st}}{\sqrt{2s} + \sqrt{2t}}\,, \quad b = \frac{\sqrt{t} - \sqrt{s}}{\sqrt{t} + \sqrt{s}}\,. \tag{34}$$

This is exactly the same solution as the one found by Alday and Maldacena in [1].

In [16], the authors found a moduli space $\{z_a, \phi\}$ for the solutions without considering the Virasoro constraints. By imposing the Virasoro constraints, we find that there are other independent relations and the solution can be fixed uniquely. This also indicates that for a general two-dimensional σ-model, where there are no Virasoro constraints, it is possible to have a larger space of solutions.

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